
Historically, finding $T$ has been cumbersome. Wasserman and Faust (1994) point out that, “...the 16 components of $T$ are difficult to calculate. There are no simple formulas. In fact, one must examine all $\binom{g}{3}$ triads, and place each into its proper category ...” (p. 567). Computationally, finding and classifying all triads in a network requires $O(N^3)$, which for large networks (many hundreds to thousands of nodes) can be quite time consuming. If one needs to calculate $T$ multiple times, as would be required to use elements of $T$ in a $p*$ framework (Wasserman and Pattison, 1996), the computational requirements are magnified.

In this paper, I present matrix formulas for calculating the 16 elements of $T$. These formulas allow one to compute $T$ without enumerating every triad and thus reduce the complexity of finding $T$ by an order of magnitude to $O(N^2)$. 2 Additionally, information contained in the matrices used to construct $T$ can be used to study the inter-relations of sub-structures in a network. For example, one can identify a network which counts the number of times actors $i$ and $j$ are linked through a structural hole White et al., 1976; Burt, 1984, such as the 201 triad.

1 The purpose of this paper is to present a more efficient way of calculating $T$. Calculating the mean, variance and covariance $T$, used to statistically test structural hypotheses under conditionally random distributions, is not changed. An example SAS-I ML program for calculating $T$ is given in the appendix. A complete set of SAS-I ML programs which calculate $T$, and test linear combinations of $T$ under the U-MAN distribution are available from the author.

2 Thanks to an anonymous reviewer for identifying the complexity of the two approaches to finding $T$: This difference can be dramatic. If it takes an $O(N^3)$ process 27 s to find $T$ in a 300 node network, and approximately 2 min in a 500-node network, it will take over 1.44 days to calculate $T$ for a 5000 node network. The comparable $O(N^2)$ procedure will take 9 s for the 300 node network, 25 s for the 500 node network, and 42 min for the 5000 node network.