9 Oscillatory dynamics of city-size distributions in world historical systems[1]

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(proof corrections; variables not italicized)

Globalization, world-system, and historical dynamic theory offer complementary perspectives for the study of city systems as the politico-economic engine of interstate networks. Here we combine these perspectives to examine a dynamical perspective on systems of cities. Globalization theory applied to Eurasia in the last millennium (e.g. Modelski and Thompson, 1996) focuses on centers of economic innovation and political power and their successive periods of rise and fall in dominance. Units of larger scale, as for example polities, are shown to operate at successively longer time-scales in their rise and fall than the economic innovation centers within those polities. World-system theory for similar regions and processes (e.g. Chase-Dunn and Hall, 1997) differs in the way in which it also focuses on innovation at the peripheries of states and empires, that is, on the marcher or boundary polities that resist the encroachment of expanding empires. Marcher states that amalgamate to defeat the spread of an empire often defeat polities formally organized on a much larger scale, thanks to cohesive or decentralized organization that is able to marshal superior combative skills or technology. World-system theory often limits itself to the more prominent types of relations, such as trade in bulk goods and interstate conflict, that form distinct macroregional networks. The structural demographic approach to the political economics of agrarian empires (e.g. Turchin, 2003, 2006) is capable of yielding a more dynamical historical account of how central polities rise and fall as their internal cohesion disintegrates with population growth into factional conflict, and of how once dominant polities and economies contend with marcher states that coalesce into formidable opponents on their frontiers.

Several of the problems in extending these kinds of complementary approaches to globalization, world-systems, and historical dynamics relate to how networks – social, political, and economic – fit into the processes of change and dynamical patterns that are observed historically. One aspect of major problems that might engage network research involves how changing network fluctuations of long-distance trade influence inter- and intra-regional dynamics. The Silk Road trade, so important in the connections through the marcher states and later empires of Mongol Central Asia between China and the Middle East, for example, also facilitated the diffusion of economic inventions from East to West that were crucial in the rise of the European city system. These included paper money, institutions of credit, and vast new knowledge, weaponry, and technologies. Spufford (2002), for example, shows how important the transmissions of innovations from China were, from a European perspective; while Temple (1987) summarizes the work of Needham (1954–2004) to show the debt owed by the West to China. A related problem, among many other open network problems in historical research is how regional and long-distance trade networks are coupled, along with conflicts and wars, to the rise and fall of cities and city systems. That is the problem we take up here.

We approach the problems of the rise and fall of commercial trade networks, regional city systems, regional conflicts, and the historical dynamics of globalization and world-system interactions in Eurasia, during the last millennium, with a concern for the valid comparative measurement of large scale phenomena. Tertius Chandler (1987) and other students of historical city sizes (Bairoch, 1988; Braudel, 1992; Modelski, 2003; Pasciuti, 2006, and others) have made it possible to compare the shapes of city-size distribution curves. These are the data we examine here in order to gage and compare the dynamics of city-system rise and fall, both within distinct regions and as changes in one region (such as China) affect changes in others (such as Europe – to give but one example).

Our approach here is to divide up Chandler's (1987) Eurasian largest city-size data into three large regions – China, Europe, and the Mid-Asian region in between – and measure variations over time that depart from the Zipfian rank-size distribution (Zipf 1949). Zipfian rank-size is the tendency for cities ranked 1 to n in size to approximate a size of M/r, where r is a city's rank compared to the largest city, and M is a maximum city size that gives the best fit for the entire distribution. This formulation allows the rank 1 largest city size S_1 to differ from its expected value under a Zipfian rank-size distribution (in which city sizes at rank k are proportional to 1/k) fitted to an extensive set of the larger cities. The Zipfian distribution has been taken to be a recurrent and possibly universal pattern for city sizes as well as many other complex system phenomena. What we find for Eurasia and regions within Eurasia is that there are systematic historical periods that show significant deviations from the Zipfian distribution. Some of these deviations show the characteristics of a regional collapse of city systems from which there is eventual recovery (unlike the cataclysmic collapse exemplified by the Mayan cities system).

The periods of rise and fall of city systems for each Eurasian region, however, are different. This allows us to test the hypothesis that the rise and fall measure for China anticipates with a time lag the rise and fall measure for Europe, which is a prediction for the period starting in CE 900 consistent with Modelski and Thompson (1996), Temple (1987) and Needham (1954–2004). Finally, for the region of China we have sufficient time-series data to test the predictions from the historical dynamics model of Turchin (2005). This allows some limited results on whether some of the same processes are operative for the rise and fall of city systems as for the historical dynamics of state and empire rise and fall.

In the first section, we pose the problem of instabilities in city sizes and systems, drawing on Chandler's data for 26 historical periods from CE 900 to 1970. In the second section, we examine ways of measuring the departure from Zipfian distributions of city sizes, and introduce the data used for city sizes and possible correlates of city-system change. In the third section we give the results of the scaling of city sizes for different regions, so as to measure city-system changes. In the fourth section, we examine the time-lagged interregional cross-correlations for these measures, and summarize the results for cross-region synchrony. In the fifth section, we examine the correlations and time lags between our three Eurasian regions, and for other variables related to known historical oscillations where we have adequate data for testing hypotheses. The variables tested include variables such as trade connectivity, internecine warfare within China, and the development of credit and currency systems that facilitate international exchange as well as innovative national markets. The final section concludes with a summary and the implications of the findings.

City-system instabilities

Jen (2005: 8–9) defines "stability" in terms of dynamical recoveries from small perturbations that return to an original state. The seriousness of the question of city-system instability and of major departures from the Zipfian derives from the assumption that city economies are organized as networks that involve trade and war, and that these economies depend on innovation to join the leading economic or political sectors of more global networks. The two main factors that make for instability are competition and population growth. Economic competition, aided by power politics, tends to make for oscillations that may return to what might be called structural stability. That is, they make for economic and political oscillations rather than conservative stationarity. Populations of polities, empires, regions, and global world systems also exhibit oscillations if we average out the trends of population growth, e.g. over the last several millennia. Jen defines "structural stability" as the ability to return from instability through dynamics other than the original (e.g. by varying external parameters) that are qualitatively similar to the original dynamic, as, for example, the standard Lotka–Voltera oscillations. While economic and political systems are not stable in the strict sense, they may have the resilience to return to structural stabilities as they pass through oscillations with differing but qualitatively similar dynamics. Major population growth trends, however, as they interact with dynamical oscillations or limit cycles, may lead to "structural instability" – an inability to return to stability even through dynamics other than the original or to recover a dynamic that is qualitatively similar to the original. The imperative of incessant competitive innovation for successful cities and city systems forms part of what leads to overgrowth of population relative to resources, and thus to subsequent system crashes. Historically, these instabilities lead eventually to industrial revolutions that, rather than conserve materials and energies, may push extravagant degradation of resources into dynamically irreversible crises such as global warming and problems of structural instabilities. Unless innovation turns toward conservation, the problems created will not be solved in the next century or possibly not in the next millennium. The issues here are ones of scale, expansions of scale (the size of cities, the size of polities and empires, and the size of economies); the dynamic interactions that operate at different scales; and how these couple spatially and temporally (as suggested, for example, in Modelski and Thompson, 1996).

The first questions of this study, then, are whether city systems as central economic actors and sites for multitudes of agents are stable or unstable, and if unstable, what kinds of models are appropriate for consistency with their dynamics. The thesis here is that it is not just individual cities that grow and decline, but entire regional (and global) city systems. Here, drawing on our earlier work (White et al., 2005), Michael Batty (2006: 592) states our case for us. "It is now clear that the evident macro-stability in such distributions" as urban rank-size or Zipfian hierarchies at different times "can mask a volatile and often turbulent micro-dynamics, in which objects can change their position or rank-order rapidly while their aggregate distribution appears quite stable" Further, "Our results destroy any notion that rank-size scaling is universal…[they] show cities and civilizations rising and falling in size at many times and on many scales." What Batty shows, using the same data as we do for historical cities (Chandler, 1987), is legions of cities in the top echelons of city rank being swept away as they are replaced by competitors, largely from other regions.[2]

Data

City-size data for historical Eurasia

Chandler's (1987) database on historical city sizes is complemented by overlapping UN population data from 1950 to the present (in the interest of brevity we do not present these results here). Chandler reconstructed urban populations from many data sources. These included areas within city walls times the number of people per unit area (see Appendix A); connected house-to-house suburbs lying outside the municipal area; data from city histories provided by city librarians; estimates from numbers of houses times the number of people per house; and the cross-checking of different estimates (see Pasciuti and Chase-Dunn, 2002). From CE 900 to 1970, his size estimates cover over 26 historical periods, usually spaced at 50-year intervals, always comprising a set of largest cities suitable for scaling in a single period. These data include 80 Chinese, 91 European, and, in between, a much larger number of Mid-Asian cities.

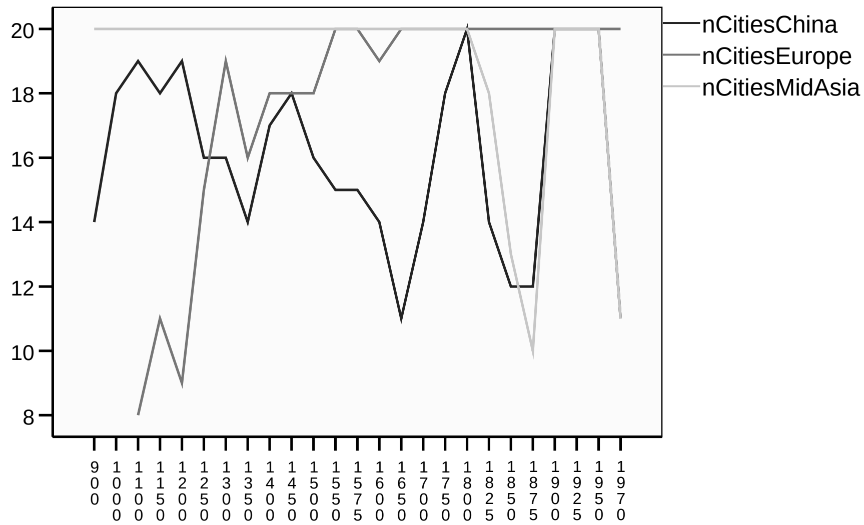

Figure 9.1 Number of Cities in Top 75 World Cities in each region when they fall below 21

Figure 9.1 shows numbers of cities in the dataset for each period when they fall below 21. European cities in the top 75 world cities increase from eight to 21 from 1100 to 1575, while those of China drop from 19 to 11 between CE 1200 and 1650. Mid-Asia has more than 20 cities in the top 75 up to 1875. One concern is whether there are too few cities in some periods to differentiate the characteristics of the tail of the size distribution (largest cities) from that part of the size distribution that reaches down to the smaller cities. Another is that of estimation from small samples.

Trade routes (Eurasia)

The total length of the Eurasian long-distance trade routes between 3500 BCE and CE 1500 at 50-year intervals have been calculated by Ciolek (2005) from trade-route maps drawn by Sherratt (2003). World-system pulsations in expansion and contraction of trade routes are shown by Turchin (2007). Turchin calculated a connectivity index for the Silk Routes between China and England. His data also show how instances of epidemics are concentrated near the high points of trade before periods of collapse.

Socio-political instability (SPI): Internecine wars (China only)

We define socio-political instability as internecine wars or regional outbreaks of social violence. Turchin (2003:164) transcribed J.S. Lee's (1931) coded five-year interval data on internecine wars in China, ranging from regional uprisings to widespread rebellions and civil wars, from 221 BCE (first unification by the Ch'in Dynasty Emperor) to 1929, in order to create a ten-year sequential intensity index to 1710. D.R. White converted Turchin's codes into a 25-year index running from 0 CE to 1700 and coded 1725–1925 directly from Lee (1931). Lee coded the period to the end of the Ming Dynasty from a remarkably systematic inventory of conflicts by Chih Shao-nan from the Tih Wang Nien Piao, and this was checked for accuracy by Tung-Kien (Lee, 1931: 114). The Tabula Annua (Seikainenkan), cross-checked and supplemented by the Tai Ping Tien Kuo Chan Ssi, proved a reliable record of Ch'in Dynasty wars, with those following 1927 from Lee's memory. The systematic pattern discussed by Lee for his graphs is one of two 800-year periods (200 BCE to CE 600, then to CE 1385) in which many more of the intra-territorial China conflicts occur in the last 400–500 years than in the early 400–300 years, and a partial repetition up to 1927 of that same pattern. The other evident pattern is for transitions between dynastic periods to be marked by more frequent and severe conflicts.

Total population and population-change data (Eurasia)

The early data on Chinese population from CE 900 to 1300 are controversial. White had data from Chao and Hsieh (1988), provided by Turchin (2003: 164–165), consulted Ho (1956, 1959), Steurmer (1980), and Heilig (1997, 2002), as well as Heilig's (1999) compilation of data from Mi Hong (1992) and Durand (1960). Given the uncertainty in absolute figures, we coded a binary variable for each 25-year period, where "1" is given for a date at which there is a population peak before collapse, with "0" otherwise. The different total population estimates available to us for China over our full time frame agreed very closely as to where these population peaks occurred. In some cases, two adjacent peaks were indicated.

Turchin (2006, 2007) provided population, carrying capacity, detrended population, and a misery index (inverse wages) for England that could be useful in comparisons with our European city data.

Monetary liquidity (China only)

For 900–1700 CE, we coded an index of monetization (liquidity) in China using Temple's (1986: 117–119) discussions (drawing on Needham 1954–2004) on the development of credit, paper money, banking, and inflation (indexing lower liquidity) into a qualitative judgmental scale from 0–10.

The q/β scaling and hypotheses

Measuring departures from Zipf's law for city-size distributions

We begin by examining the city-size distributions of macroregions in the Eurasian continent, over roughly the last millennium, which is the millennium of Eurasian and then planetary globalization. We examine the extent of instability of city systems in ways that are visually evident by inspection of changes in the shapes of Eurasian city-size distributions,

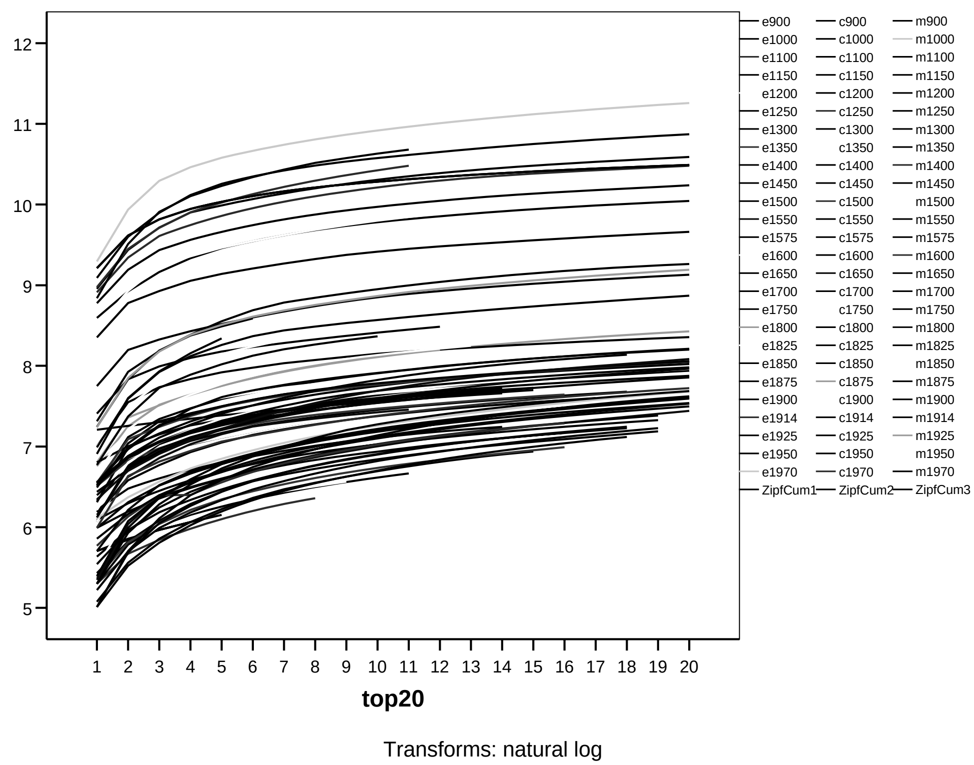

Figure 9.2 The Chandler rank-size city data (semi-log) for Eurasia (Europe, China, Mid-Asia). Some lighter contrastive curves are not visible. Original in color at http://intersci.ss.uci.edu/wiki/index.php/Tsallis_q_historical_cities_and_city-sizes.

and as measured by shape indices. Figure 9.2 shows a semi-log graph of the cumulative rank-size distribution for divisions of most of Eurasia (excluding Japan/Korea) into three regions: China (c.900–c.1970), Europe (c.900–1970) and the Mid-Asian (c.900–c.1970) remainder. The curve to which power-law distributions should correspond is shown by the top (ZipfCum) power-law curve. Cumulative population size is logged on the y-axis, and the x-axis is city-size rank. The Zipfian curve forms a straight line when the rank is also logged, with a Pareto (1896) log–log slope of 2. As can be seen visually, there is some departure from the perfect parallelism in the empirical curves: some lines are more curved or less curved for the top cities than the Zipfian; most lines are flatter than the Zipfian for the smaller cities; and many of the curves bend at different city ranks.

Figure 9.3 shows the same data (all three regions) in a log–log plot where power-law city distributions would all be straight lines and Zipfian distributions would all have the same slope. The lines are neither parallel nor of the same slope, and neither do they have curvatures in the same places. Our measurements of the properties of these distributions will be aimed at the hypothesis that these variations provide indicators useful for showing how city-system fluctuations fit into economic and historical dynamics.

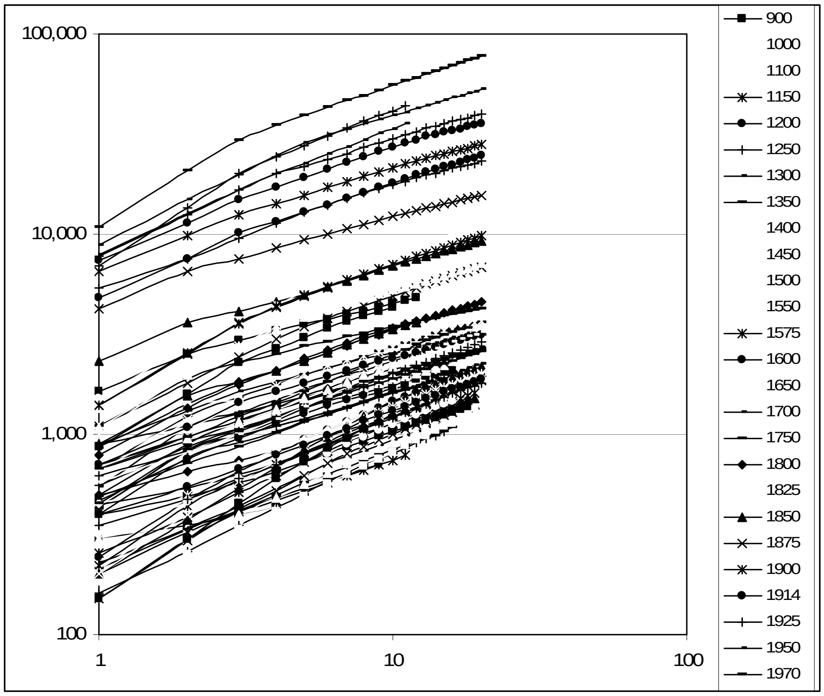

Figure 9.3 The Chandler top 20 rank-size city data (log–log) for Eurasia (Europe, China, and Mid-Asia). Europe legend shown, China and Mid-Asia legends not shown. Some lighter contrastive curves are not visible. Original in color at http://intersci.ss.uci.edu/wiki/index.php/Tsallis_q_historical_cities_and_city-sizes.

We use two measures of how these curves vary. Each measurement model is based on a standard continuous function with parameter fitting based on recasting the data as a cumulative probability in its complementary form. $P(X \geq x)$ is the probability that an urbanite will reside in a city of at least size x. Each model is aimed at capturing the complete (Pareto II) shape of the empirical $P(X \geq x)$ or part of that shape (Pareto I for the tail) as a cumulative complementary distribution function (CCDF). It is important to use maximal likelihood estimates (MLE) in fitting the parameters of distributional models, and especially so with small samples. MLE is consistent and both asymptotically normal and unbiased, the latter meaning that with n independent samples of the same data, the expected values of the estimated parameters for each sample converge more closely to the true parameter values the larger the total sample (Clauset, Shalizi, and Newman 2007).

The first measure is the standard (type I) Pareto distribution with a single parameter β where the Zipfian is the special case of β = 2:

$$P_\beta(X \geq x) \sim x^{-\beta} \qquad (1)$$

The fit to this distribution captures the extent to which the lines are straight in Figure 9.3, as is typical for the tails of city-size distributions.

Fit to a second function, a type-II generalized Pareto distribution, with a scale σ (Arnold, 1983), captures the extent to which a given log–log distribution is curved. This function allows the recognition that Zipf's law and power laws for city sizes almost always have one or more cut-offs of a lower size below which the power-law changes or ceases to apply.[3] This function fits a shape parameter θ (analogous to β), and a scale parameter σ, to the urban distribution.

$$P_{\theta,\sigma}(X \geq x) \sim (1+x/\sigma)^{-\theta} \tag{2}$$

The type-II Pareto has an extra scale parameter σ, but given sufficiently large samples and maximal likelihood estimation (MLE) of parameters, there are many advantages to a parameter that specifies the cross-over where the curve breaks for a power-law or Zipfian tail. The Pareto II function:

1 can fit an exponential function or a collapsed tail in cases where a power-law or Zipfian tail is inapplicable.
2 with MLE (maximal likelihood estimation), yields parameter estimates that are consistent and asymptotically unbiased. MLE runs can be batched for multiple datasets, and the MLE commands are easy to copy and paste into R.[4] Standard errors are typically very small for city-size data, and the true values of the parameters are likely to be within these limits.

MLE provides consistent parameter estimates that allow one-to-one functional transformations of the parameters into equivalent models (Brown, 1986). The q-exponential $Y_q(S \geq x) = Y0(1 - (1 - q)x/\kappa)^{1/(1-q)}$ and its shape (q) and crossover (κ – i.e. kappa) parameters is a one-to-one functional transformation of equation (2) where $q = 1 + 1/\theta$ and $\kappa = \sigma/\theta$. Y_q has been previously used (Malacarne et al., 2001) for city-size distribution scaling. Bootstrap estimates of the standard error and confidence limits of the q,κ parameters derived from θ,σ are provided by Shalizi's (2007) R program for MLE. There are many other advantages of Y_q and Pareto II that we do not exploit here.[5]

A continuity theorem often used with the Y_q model (Tsallis, 1988) gives the relation in the Pareto II and Y_q functions, when $1 < q \leq 2$, between a Y_q tail that asymptotes to a power law with slope $1/(q - 1)$ and a Pareto I tail with slope β. The Y_q curvature, however, captures that of the smaller cities in the distribution. The theorem is important to the study of city sizes where smaller cities do not follow a power law and we often lack data on the actual distribution for smaller cities – a problem that we will take up next. More generally, although that is beyond the scope of this chapter, the q-exponential is interpretable as one of the theories developed in "behavioral statistical physics"

(Farmer, 2007) and $q = 1$ (the limiting case of an infinite θ) has a special meaning as an entropic system "at rest," lacking

complex interactions and power-law behaviors. For cities, $q \leq 1$ is an indicator of system collapse, while for $q > 2$ the larger cities are size outliers that disrupt the estimation of variance.

Measurement error for Pareto II departures from Zipf's law for city-size distributions

There are two potential sources of error for Pareto II parameters that are not soluble without MLE. They are sufficiently important to merit discussion here, because first they affect the accuracy and interpretation of results, and second they provide crucial information needed for others to replicate results using MLE. One is the boundary specification of the region studied. China and Europe, for example, are more naturally bounded areas than our Mid-Asia, which runs from Cordoba to India and Southeast Asia and is partly defined by the spread of Islam. The second is the size of the sample: if there is a small number of largest cities for which there are data, estimation of the scale or crossover parameter $\sigma = \kappa/(q - 1)$ of the Pareto II distribution may err because the sample does not provide the relevant evidence for curvature away from a power-law tail. In this case σ will be close to zero to insure an approximation to the ordinary Pareto because the 1 in $1+x/\sigma$ from equation 2 becomes negligible when the values of x are divided by a diminishingly small σ (i.e. they are multiplied by a large constant). In China and Europe, we find a strong coefficient of determination (significant at $p < 0.005$) between $\sigma < 0.04$ and having fewer than 18 cities in the sample. In this case, σ is commonly in the range $0.00001 > \sigma > 0.0000001$, as contrasted to $\sigma > 5$ (ranging up into the thousands) for the larger city samples. Must we conclude when σ is small, in the cases of small samples of the largest cities, that the appropriate distribution function is Pareto I and not Pareto II (q-exponential) when the Pareto I fit may be due to missing data? These small values of $\sigma(\ll 1)$ are non-arbitrary, however, and the fitted Pareto I parameter $\beta(\approx\theta$ in this case) must converge to $1/(q-1)$ when $1 < q \leq 2$.

We are thankful to Shalizi (2007) for writing an MLE program in R for our use in estimating parameters θ,σ (Arnold, 1983) in equation (2).[6] He also gave us his R program for standard MLE fits to the Pareto distribution.

The left-hand graph of Figure 9.4 shows some of the early, middle and late period q-exponential curves for Chinese cities fitted by MLE using a normalized cumulative probability distribution. These are for periods where $\sigma > 1$ (i.e. $\kappa > 1/\theta \approx q-1$). These cases also fit the q-exponential better than a power-law MLE for the same data. What this graph shows is a very regular pattern for the fitted curves. First, like the empirical data, the fitted curves tend to asymptote toward a power-law (Pareto I) tail for the larger cities. Second, there is considerable variation in the slopes of these tails. Third, there is a second horizontal asymptote convergent on $P(X \geq x) = 1$, where "cities proper" cease to occur at smaller settlement sizes. This allows an estimate and reconstruction of total urban population at different city sizes. Fourth, there are distinct historical periods, ones that come in three clumps, within

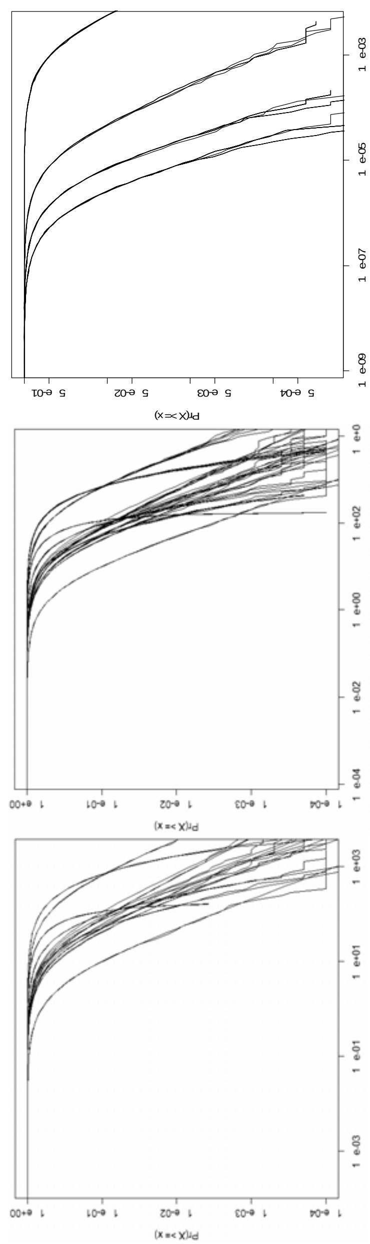

Figure 9.4 Probability distributions of Chandler rank-size city data (log–log) for China. Smooth lines are for the fitted curves in successive time periods, and the jagged lines are illustrative bootstrap sample fitting used to estimate error bounds.

which there are ups and downs in slopes. Fifth, each period tends to have something of a fixed pivot (shown by the arrow) below the cross-over from the power-law tail to smaller sizes. The center graph in Figure 9.4 shows the same Pareto II curves, but with additional fits for random samplings of the same data points. Additional fittings like these allow bootstrap estimates of the standard errors of estimates, which in almost all our empirical cases are very small.

The right-hand graph in Figure 9.4 shows the normalized probability MLE fits for China in historical periods where values of σ are small («1).

Small σ may be caused by small samples, as seen in many periods in Figure 9.1 for China or Europe, or by summation of distributions from different regions (as with Mid-Asia). The precision of ML parameter estimation solves the problem of comparability of q values given very different σ estimates («1 and > 1), but relabels the x axis in the σ « 1 case on a scale that allows us to focus on the small values of σ but also to see the same shapes of the Pareto II distribution as in the cases where σ >1.[7] The bootstrap standard errors for σ and κ are small within each period (but not across periods). The σ « 1 values are thus technically correct, along with q, although precision may be added in further studies through MLE correction for the very smallest sample sizes, since they are more likely to be underestimated.

The Mid-Asian region has more cases where σ « 1, but these are neither caused by nor correlated with small samples. Rather, the broad Mid-Asian power-law tails are likely to result from summing somewhat divergent regions that each have shorter power-law segments in their tails, with summation making for a longer Pareto tail in the aggregate (Farmer, 2007). What this indicates is that the magnitude of the θ parameters for Pareto II might be underestimated (longer tails with lower θs than the true values for properly specified regions).[8] Since our comparisons depend on relative variations within or across regions, these might not affect our findings on Mid-Asia.

The MLE methods, then, solve many but not all the potential measurement errors for Pareto II departures from Zipf's law. Remarkably, when only the power-law tail is present in the sample, without any overt indication of departures from the power-law, MLE still estimates q correctly in a way that converges with a power-law fit for the tail, but it retains the prediction that the fitted q and κ, in the normalized probability distribution, represent a consistent prediction of both smaller city sizes and total population.

Hypotheses

Several linked hypotheses build on one another, each supposing the previous hypotheses to be supported, and each adding greater specificity in relation to the parameters of the two models, β for the Pareto I and θ for Pareto II (thus q for the q-exponential):

H1 The Zipfian (β =2 and q=1.5 for our CDF) is posited as the most likely historical norm for the tails (β) and bodies (q) of city-size distributions.[9]

Over long historical periods these should be expected as average values around which q and β fluctuate.

H2 Variations in q and β are conservative as population measures that are affected by births and normal mortality, but may change quickly when influenced by migration, and by socio-political instability (SPI), that is, internecine wars or outbreaks of violence.

H3 Variations in q and β are thus likely to exhibit stability within historical periods of multiple generations, with Zipfian values on both measures correlated with periods of stability and normality, followed by instabilities that may occur suddenly.

H4 As such, q and β are indicators of rise and fall of urban-system size distributions in both the body and the tail of the distributions, which may vary with considerable independence.

Tails may (a) collapse and shorten (β dropping toward 1), or grow into (b) thicker, shallower (β ≤ 2) sloped tails, (c) Zipfian (β ≈ 2) tails, or into (d) thinner, steeper (β ≥ 2) sloped tails.

(4a) Collapse of tails should co-occur with socio-political instability (SPI), severe economic/political crisis, or major wars.

(4b) Longer tails should be enhanced by the capitals of empires and by exceptional centers of international trade that serve as economic magnets for migration from impoverished rural areas.

(4c) Zipfian tails should be enhanced by intra-regional trade with positive urban-hierarchy feedbacks.

(4d) Shorter tails should correlate with external conquest of a capital or of hub cities.

H5 Intra-regional and inter-regional trade is crucial for city-system rise, and economic collapse may be involved in city-system collapse. Fluctuations of inter-regional trade may act either or both to synchronize city rise and fall between regions, or to predict from city rise and fall in a more developed region that time-delayed rise and fall will occur in a less developed region that is strongly connected by trade. Currency, credit, banking, and liquidity are leading indices of development in measuring the inter-regional impact of trade.

H6 Historical phases that are clearly marked in the population/instability cycles of structural demographic historical dynamics for periods in which agrarian empires that are relatively self-contained, lacking external perturbations (Turchin, 2003, 2005, 2006), should correlate with some but not necessarily all of the phases of city-system rise and fall, especially those involving SPI fluctuation. In this context, population growth with low pressure on resources should lead to rises in q and β, provided that trade and liquidity allow economic elites to congregate in cities and to provide employment for skilled workers. Peaks of population pressure on resources followed subsequently by high SPI levels should precipitate city-system declines. In these terms, city-system rises and falls:

(6a) are likely to be loosely but not strictly coupled into structural demographic variations.

(6b) are also interactive with inter-regional competition and global wars (Modelski and Thompson, 1996).

(6c) are likely to exhibit both longer and shorter (i.e. less predictable) historical periods of stability, given this combination of regionally endogenous and regionally exogenous dynamics.

Those hypotheses are testable. The following are more speculative observations about evolutionary tendencies, predicated on our findings:

H7 In spite of the growth of gross world product (GWP) rising at faster rates than population, there is no evolutionary historical tendency evident toward either greater stability, greater instability, or alteration in the periodicities of city-system ups and downs, in spite of, and probably because of, the pressures of rising global and rising urban populations. Evolutionary stability in city systems, as a form of recovery from small perturbations, apparently remains something to be learned rather than taken for granted.

H8 Stability as recovery from large perturbations through structural demographic oscillatory dynamics, however, has apparently been learned, but urban instabilities are only weakly coupled to oscillatory structural demographic recoveries, so evolutionary learning is only partial. The introduction of new dynamics complicates the question of whether city systems will acquire greater stability.

Scaling results

Using visual and statistical evidence for changes in the shape parameter, White et al. (2005) in an earlier study were able to date six Q-periods in Eurasia over the last millennial period. These changes and periods were seen to be related to the framework for studying globalization developed by Modelski and Thompson (1996). In studying multiple regions, we take a more detailed and dynamical view of this relationship.

Figure 9.5 shows the q and β slope parameters fitted by MLE for the regions of China, Europe, and the region between (Mid-Asia, from the Middle East to India). Here, a Zipfian tail would have $q = 1.5$ and $\beta = 2$. The horizontal line shows that this slope and shape is approximated more recently in the early modern and modern period. We also show a normalized minimum of q and β, in which we divide q by 1.5 and β by 2.0 to normalize for the Zipfian. Table 9.1 summarizes all the descriptive statistics used.

These results support H1, that the Zipfian is the historical norm both for tails and bodies of city size distributions, approximating β=2 and q=1.5. Mean

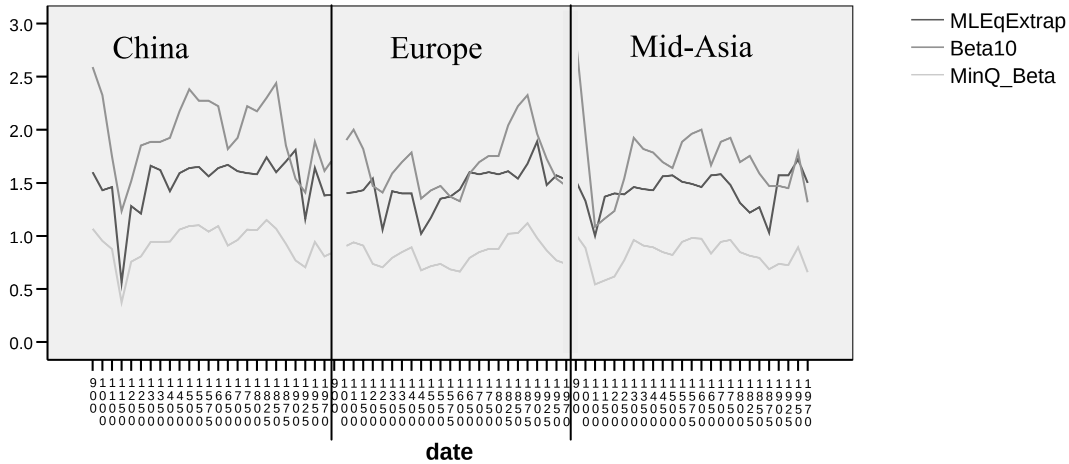

Figure 9.5 Values of q, beta, and their normalized minima

Table 9.1 Descriptive statistics for city curve shapes

	N	Minimum	Maximum	Mean	Standard deviation	Standard deviation/ Mean
MLEqChinaExtrap	25	.56	1.81	1.5120	.25475	.16849
MLEqEuropeExtrap	23	1.02	1.89	1.4637	.19358	.13225
MLEqMidAsIndia	25	1.00	1.72	1.4300	.16763	.11722
BetaTop10China	23	1.23	2.59	1.9744	.35334	.17896
BetaTop10Eur	23	1.33	2.33	1.6971	.27679	.16310
BetaTop10MidAsia	25	1.09	2.86	1.7022	.35392	.20792
MinQ_BetaChina	25	.37	1.16	.9645	.16247	.16845
MinQ_BetaEurope	23	.68	1.26	.9049	.15178	.16773
MinQ_BetaMidAsia	25	.54	1.01	.8252	.13217	.16017
Valid N (listwise)	19					

values for q in the three regions vary around $q = 1.5 \pm 0.07$, consistent with a Zipfian tail, and similarly for variations around $\beta = 2 \pm 0.07$ for the Pareto slope of the top 10 cities. Statistical runs tests of whether the variations around the means are random or patterned into larger temporal periods are shown in Tables 9.2 and 9.3. The runs tests reject the null hypothesis ($p < 0.01$ for Europe, $p < 0.05$ for Mid-Asia, and $p < 0.06$ for China; $p < 0.00003$ overall), supporting H3.

The time periods of successive values above and below the medians represent the rise and fall of q to Zipfian or steeper-than-Zipfian slopes alternating with low-q periods with truncated tails of the distributions. Relatively long city-slump periods occur in the medieval period for all three regions, then a second slump occurs in Europe in 1450–1500, another in Mid-Asia in 1800–1850, and one in China in 1925 (not shown) when q falls to 1.02.

Table 9.2: Runs Tests at medians across all three regions

	MLE-*q*	Beta10	Min(*q*/1.5, Beta/2)
Test value(a)	1.51	1.79	.88
Cases < Test value	35	36	35
Cases >= Test Value	36	37	38
Total Cases	71	73	73
Number of Runs	20	22	22
Z	-3.944	-3.653	-3.645
Asymp. Sig. (2-tailed)	.0001	.0003	.0003

Table 9.3: Runs Test for temporal variations of *q* in the three regions

	mle_Europe	mle_MidAsi	mle_China
Test Value(a)	1.4	1.4	1
Cases < Test Value	9	11	1
Cases >= Test Value	9	11	1
Total Cases	18	22	2
Number of Runs	4	7	7
Z	-	-	-
Asymp. Sig. (2-	.00	.04	.

a Median

As for H6, there are rough correlations for both secular cycles (Turchin, 2003, 2005, 2006, 2007) and Modelski and Thompson (1996) globalization processes with the dates of urban crashes, shown in Table 9.4 below.

As shown by the lower dashed line in Figure 9.6, values of q below 1.3 might be considered as reflecting a city-system crash or collapse/destruction of the largest cities. China and Europe experience an abnormal rise in q in

Table 9.4 Temporal breaks and urban crashes of β/*q* in the three regions

Breaks	950	1150	1430	1640	1850	
Cycle	1	2	3	4	5 (Modelski-Thompson 1996: Table 8.3)	
Mid-Asia:	**1100,**		1450		**1825-75, 1914**	**(major**/minor **urban crashes)**
China:	**1150-1250,**			1650	**1925**	**(major**/minor **urban crashes)**
Europe:		**1250,**	**1450-1500,**		1950?	**(major**/minor **crashes)**

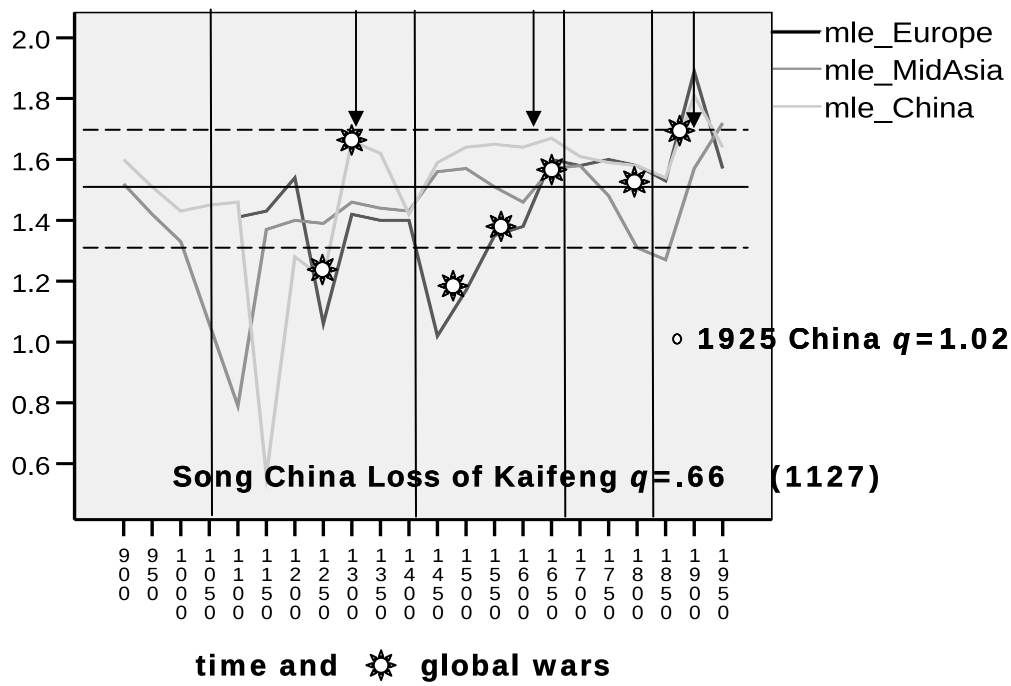

Figure 9.6 Fitted q parameters for Europe, Mid-Asia, and China, CE 900–1970, with 50-year lags. Vertical lines show approximate breaks between Turchin's secular cycles for China and Europe. Downward arrows represent crises of the fourteenth, seventeenth, and twentieth centuries

1900 beyond 1.7 (upper dotted line). This results in a thick-tailed distribution (extreme primate cities) that might be considered as a different kind of city-system crisis. Some crashes have to do with wars, like the Song's loss of their capital to the Jin in 1127. Global wars, noted by stars on the lines in Figure 9.6, might be connected with the punctuations of these periods, but we are unable to evaluate that question statistically. Crashes in q often occur at long intervals (**bold** dates in Table 9.4), as in Figure 9.5, with β falling at shorter intervals.

These results support H2, that variations in q and β are conservative (since births and normal mortality are slow to affect population measures) but may also change quickly in ways consistent with inter-urban migration, internecine wars and outbreaks of violence or general socio-political instability (SPI).

They also support H3, that variations in q and β may have long periods of stability, with Zipfian values on both measures correlated with periods of stability and normality, and that stability may be followed by sudden instabilities, or drops in q, β, or both.

Cross-correlation of the scaling measures

One of the major patterns of variability in city distributions is the primate city effect: the primate and top-ranked cities often form a steeper urban hierarchy in periods of economic boom or empire, or when the primate city is a major international trade center. In periods of decline they may form a truncated tail compared with the body of the distribution. Further, the slope of the tail of the size distribution (β) tends to change faster than the body shape of the distributions. This is tested using data from all three regions using the autocorrelation function (AFC), where values of a variable in one time period are correlated its values for 1–16 time lags (in this case each lag being 50 years). The upper and lower confidence limits are at 95 per cent for a two-tailed significance test ($p < 0.05$). The AFC of β compared with q shows a much higher short-term continuity (one lag of 50 years), a recovery period at five to six lags, and then autocorrelation largely disappears, while q varies more continuously with more stable long-term autocorrelations (up to 16 lags or 800 years). The ratio of q/β has autocorrelation only for Europe, and is oscillatory but the autocorrelation is not statistically significant.

Figure 9.5 has shown that q and β vary somewhat independently, often correlated positively when $\sigma \ll 1$ ($\kappa \ll 1$), recalling that β is a negative slope and q varies inversely to that slope but negatively when $\sigma > 1$ ($\kappa > 1$). But which one affects which over time if the two are synchronously somewhat independent? In a time-lagged correlation: does the shape (q) of the body of the city affect the tail (β) in subsequent periods, or the reverse? The β might shape q if long-distance trade has an effect on the larger cities engaged in international trade, but q might shape β if it is the waxing and waning of industries in the smaller cities that feed into the export products for the larger cities, as we often see in China and Europe.

What lagged cross-correlations show for China and Europe – but not in Mid-Asia – is that, starting from the maximal correlation at lag 0, high q (e.g. over 1.5) predicts falls in Pareto β over time, reducing the slope of the power-law tail below that of the Zipfian. This suggests that high q produces an urban system decline in β. This would contradict a hypothesis of long distance trade as a driver of rise and fall in the larger cities. However, it would not contradict the possibility that long-distance trade was directly beneficial to the smaller cities, with these effects feeding into the success of the larger cities but with a time lag. For China and Europe, where successful long distance trade was organized on the basis of the diffusion of effective credit mechanisms available to the smaller merchant cities, this seems a plausible explanation for the time-lag findings. These credit mechanisms were not so easily available in Mid-Asia where Islam operated to regulate interest rates to prevent excessive usury.

If there is a correlation between long-distance trade and the rise and fall of population pressure in the secular cycles of agrarian empires, our data might support Turchin's (2007) argument, formulated partly in response to our own studies of the role of trade networks in civilizational dynamics, that it is during the high-pressure (stagflation) period that long-distance trade flourishes. If so, then the impact of trade should be reflected first in variations in q, which vary more slowly than β.

The overall pattern in the cross-correlations for the three regions together shows strong correlation synchronically between q and β at lag 0 ($p < 0.000001$), where high values of q predict falling values of β over time over three 50-year lags. Variables q and β have the least cross-correlation for Mid-Asia, but detailed examination of the Mid-Asia time lags shows a weak cyclical dynamic of Hi-q→Lo-β →Lo-q→Hi-β that holds up to 1950.

Historical network and interaction processes

H5 posited that intra-regional and inter-regional trade is crucial for city system rise, and economic collapse may be involved in city-system collapse. The data presented here support this hypothesis, but we approach the question first as to whether, at the inter-regional level, there is synchrony between regions, and of what sort, and whether trade is involved in this synchrony.

Turchin (2007) shows evidence for “a great degree of synchrony between the secular cycles in Europe and China during two periods: (1) around the beginning of the Common Era and (2) during the second millennium.”

We also find evidence for synchrony in city-system rise and fall in common temporal variations in q for the second millennium. The correlations in q by time period follow a single-factor model, as shown in Table 9.5, with China contributing the most to the 47 per cent common variance in q between the three regions.

The evidence from city sizes adds detail on dynamical interaction to that of inter-regional synchrony for the last millennium, supporting H5. Figure 9.7 shows that changes in q for Mid-Asia lead those of China by 50 years, with a hugely significant correlation at 50-year lag 1; Mid-Asia leads Europe by 150 years (lag 3) but the cross-correlation is not quite significant. The cross correlation for China’s q leading Europe by about 100 years (lag 2) is also not quite significant, although several earlier estimates of q did show significance (recall that the true values of q may vary somewhat even for MLE estimates).

H5 posits the historical specificity that Eurasian synchrony has been partly due to trade, particularly that between China and Europe (also noting that the practice of Islam in the Mid-Asian region during this period tended to restrict the full employment of credit mechanisms). The cross-correlation in Figure 9.8, showing an effect on the growth of β in Europe, sustained by the Silk Road trade, for example, suggests that trade is one of the factors causing the growth of power-law tails in urban size distributions, again supporting H5.

From European data contributed by Turchin, Figure 9.9 shows that there was synchrony between the higher values of q (normal urban hierarchy) and the percentage of the French population attracted to Paris as a regional capital and economic center, with this percentage falling after the peak in q.

Our choice of the last millennium to test the interaction of the city size fluctuations with historical dynamics was motivated by the evolution

Table 9.5 Principal component single-factor analysis of contemporaneous regional values of q

Communalities

	Initial	Extraction
MLEChina	1.000	.660
MLEEurope	1.000	.444
MLEMidAsIndia	1.000	.318

Extraction Method: Principal Component Analysis.

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.422	47.403	47.403	1.422	47.403	47.403
2	.944	31.467	78.870			
3	.634	21.130	100.000			

Component Matrix

	Component 1
MLEChina	.812
MLEEurope	.667
MLEMidAsIndia	.564

of globalization in Eurasia in this millennium. Key elements in the transition to market-driven globalization occurred in China, starting in the period of the tenth-century invention of national markets, with currencies, banks, and market pricing – a historical sequence that leads, through diffusion and competition, to the global system of today (Modelski and Thompson, 1996).

The data on credit and monetary liquidity in the Chinese economy also closely follows the rise and fall of q, as shown crudely in Figure 9.10, supporting H5. The rise of monetization, the growth of credit, and the development of banking accompany the early Zipfian q ≈ 1.5 of Song China, and these mechanisms of liquidity plummet with the Jin conquest of Kaifeng. The c.700–800 years from 1100 CE, with long periods of inflation, are required to regain liquidity and banking favoring international trade. During the Qing dynasty the Chinese money was silver coin. The first modern bank, the Rishengchang (Ri Sheng Chang) was established in 1824. It broadened to include banks in every major city, folding in bankruptcy in 1932.

For further tests of H6, we have scant data on total population relative to resources, and we have reliable data for the last millennium only for England in comparison with our Eurasian city data. There are few points of comparison, but temporal synchronies appear in those few points: 1300 and

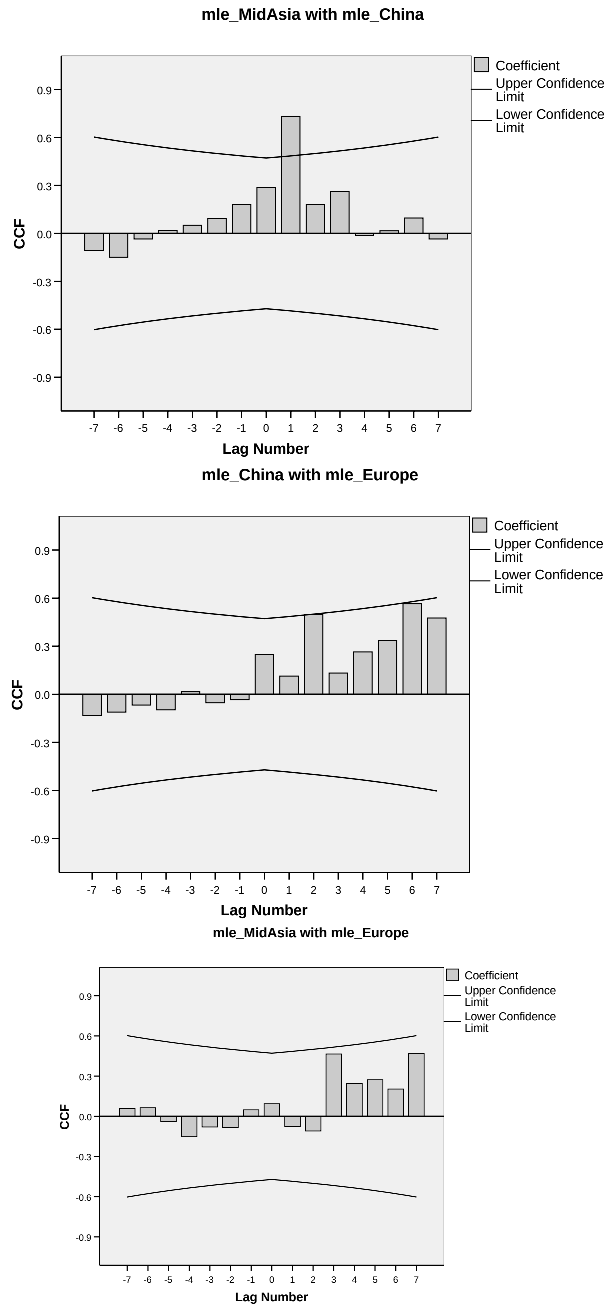

Figure 9.7 Cross-correlations for the temporal effects of one region on another

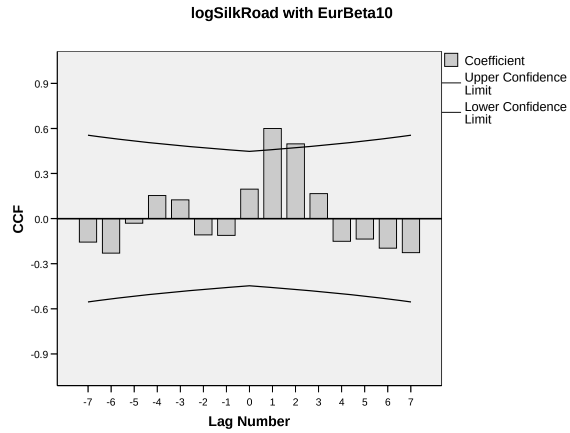

Figure 9.8 Time-lagged cross-correlation effects of the Silk Road trade on Europe

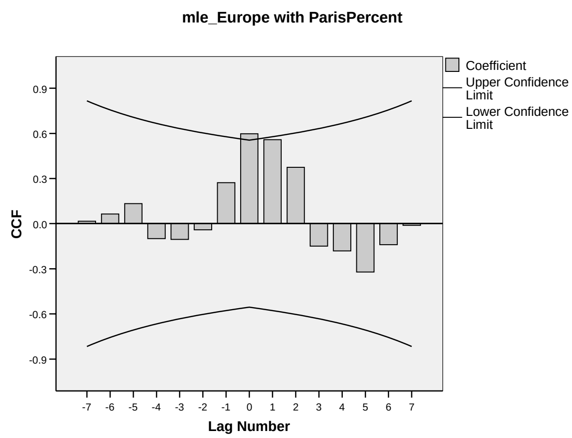

Figure 9.9 One time-lagged effect of regional q on a primate city population

1625 are the peaks of scarcity for ratios of people to resources, and pre-1100, 1450, and 1750 are the troughs of plentiful resources. The peaks of scarcity correspond with slumps in q and the troughs to rises. It is impossible to rule out at this point the possibility suggested in H6 that the urban system fluctuations that we observe are interactively linked to Turchin's secular cycles, particularly if we include both types of fluctuations: those in q, in β, and in our normalized minimum of the two, as well, which may reflect either type of slump.

Figure 9.11 shows support for hypothesis H6 – the coupling between urban system dynamics and structural demographic historical cycles (Turchin, 2005). The three insets show Lee's (1931) data on socio-political instability (SPI: specifically, internecine wars) for the periods of the Han, Tang, and

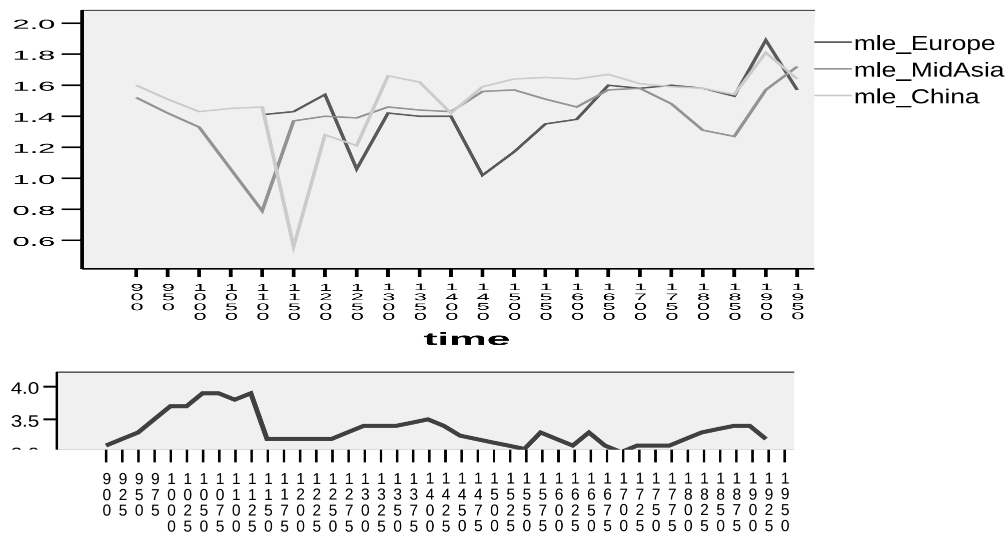

Figure 9.10 A crude long-term correlation between Chinese credit and liquidity (lower graph), estimated from Temple (1986), and Chinese (European, Mid-Asian) data and q (upper graph)

Song dynasties (broken lines) against data on detrended population (solid lines).* J. S. Lee interpreted his data on Chinese internecine wars from 200 BCE (Han period) to 1930 as showing 800-year cycles of internecine conflict, weakly separated into two 400-year periods. Figure 9.11 breaks up his data into agrarian empire historical periods in which there are no major disruptive external wars. The relation of population pressure phasing to that of SPI is similar in each case, with SPI lagging population pressure P** at a roughly generational interval. This produces four phases: population growth (and certain types of innovation) in phase 1; rising resource scarcity and SPI, rising to a population peak in the stagnation phase, phase 2; a phase 3 of falling SPI and falling population pressure; and a fourth phase with minimum population pressure and SPI. Each period repeats a similar endogenous dynamics defined by the negative time-lagged feedback between P, population density per resource, and SPI, fitting a two-equation model (with appropriate constants):

$SPI_t \sim A\, P_{t-1}$ (population change drives change in SPI) (3)

$P_t \sim -B\, SPI_{t-1}$ (SPI has a severe effect on Population) (4)

The third inset in Figure 9.11 falls within our period of study for China and adds our measures of city-system parameters q (diamonds) and β (thin line) each multiplied by 10 for easy visibility.

*Han and Tang regional populations are divided by the estimated agricultural food supply (Turchin 2005); while the Ming regional population (Zhao and Xie 1988) is divided by the growth trend.

** Here we use our 0/1 population coding defined on page 197.

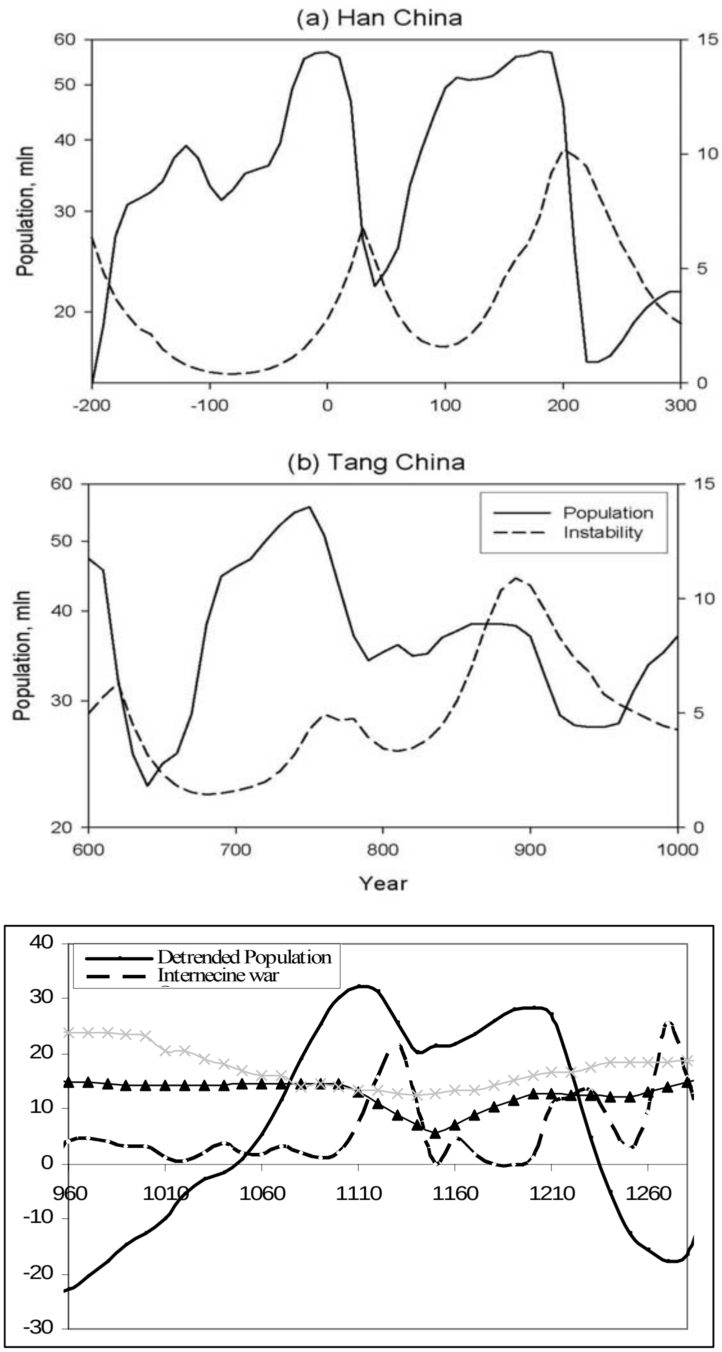

Figure 9.11 China's interactive dynamics of socio-political instability (broken curve for internecine wars – J. S. Lee 1931) and population (solid curve): figures (a) Han (200 BCE to CE 300) and (b) Tang (CE 600 to 1000) from Turchin (2005), with population detrended by bushels of grain; (c) Song Dynasty population (CE 960 to 1279) divided by successive trend values

The fall in β (see the thin line) in Figure 9.11 (c), which indexes a fatter tail for large cities, caused for example by people leaving the largest cities to go to smaller ones, occurs with a rise in population pressure. Decline in q (triangles) signals a change in the shape of the city-size curve that affects the smaller cities and works to their advantage in indexing the disproportional change in size of smaller cities "up" into the thicker tail of the city-size distribution: this occurs in Figure 9.11(c) just before the population peak. The trough of β co-occurs with the peak of the SPI crisis of socio-political violence. The trough of q occurs once the SPI crisis has ended, followed by rising q with renewed growth in population pressure, P, then leveling of q before the next population peak. As in the endogenous dynamic of sustained fluctuation modeled by Turchin's equations (1) and (2), upward P leads upward SPI by a generation and downward SPI leads upward P by a generation.

A four-variable dynamic including q and β, however, is not that simple. Two-equation time-lagged regressions with constants fitted to each term behave similarly for China and Europe, 925–1970, as shown in equations (5) and (6), except that the SPI index affects q without a time lag.

Socio-political instability tends to have an immediate effect on the relation between smaller and larger cities that affects q but not β:

$$\beta_t \sim -C\, q_{t-1} + q_{t-1}\beta_{t-1}$$

$$\text{(overall } R^2 \approx 0.79\text{, China } R^2 \approx 0.75\text{, Europe } R^2 \approx 0.69) \qquad (5)$$

$$q_t \sim -D\, \beta_{t-1} + q_{t-1}\beta_{t-1} - SPI_t$$

$$\text{(overall } R^2 \approx 0.57\text{, China } R^2 \approx 0.54\text{, Europe } R^2 \approx 0.66) \qquad (6)$$

Further, without the effect of SPI, these two equations, unlike (1) and (2), would predict positive feedback between β and q that would result in either a convergent or a divergent time series.† It is only the SPI index, given the locations of SPI events (which we also estimated for Europe from historical data) with the Turchin dynamics that acts as external shocks which make the predicted time series oscillatory, and often synchronous with Turchin's endogenous dynamic (modeled by equations 1 and 2). Significantly, then, there is support for the idea that it is the conflict events within Turchin's endogenous dynamic that drive q in the city dynamic, which in turn drives β (equation 3) in that dynamic. Figure 9.11(c) represents the case for Song China, where q and β are more stable over time than population pressure and SPI, but their periods of collapse are affected synchronically with generational time lag according to the dynamics of Turchin's model of agrarian empires.

Also significant, the three agrarian empire periods (Han, Tang, and Song) in Figure 9.11 are separated by periods of external wars and instabilities that

†A two-equation reciprocal time-lag model, such as that given in equations (3) and (4) produces fluctuations if the signs of the right-hand elements are opposite, but convergence or divergence if they are the same. This can be verified in difference equations using initial values that generate a full time series.

interrupt the endogenous dynamics between population pressure and SPI fluctuations. Each period resumes fluctuation modeled by the two-equation time-lagged dynamics of P and SPI, which tend to obtain only when major external disturbances are absent. The external wars in Lee's (1931) analysis show up as large SPI fluctuations between dynasties (usually leading to their termination and a later successor empire), while internal SPI fluctuations occur within the periods of relatively high endogeneity. These various cycles couple to form the larger 800-year cycles of multiple successive empires, observed by Lee, marked by the most violent transitions. Overall, there is support in the Chinese and European examples for the aspects of hypothesis H6 initially designated 6a, 6b, and 6c.

Strong evidence for the coupling of urban system dynamics with population/instability (structural demography) historical dynamics is presented in Figure 9.12. Much as SPI leads population declines in historical dynamics, the

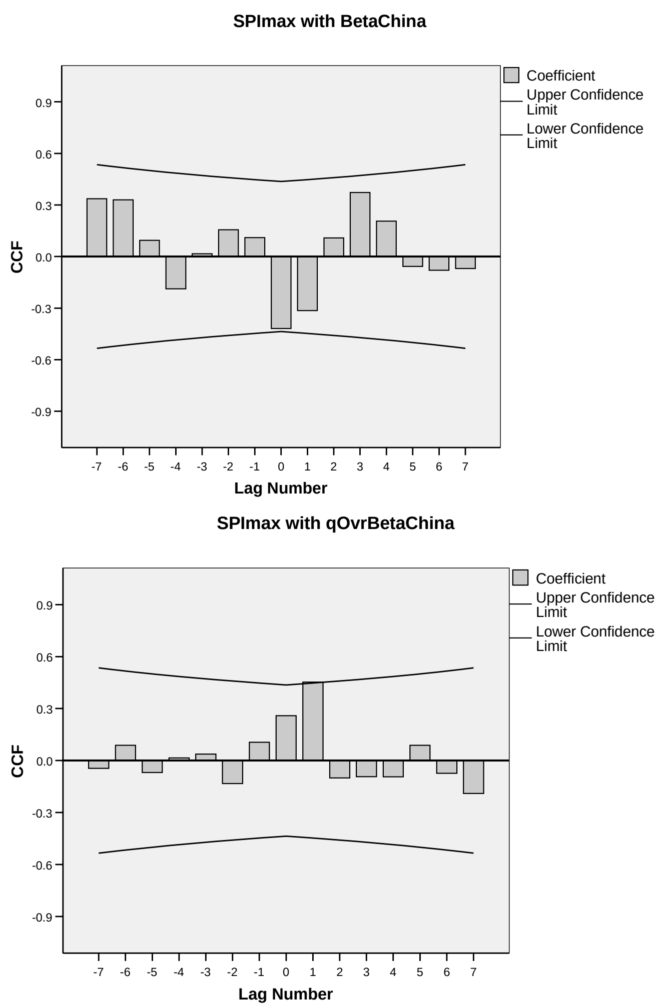

Figure 9.12 Cross-correlations of q and β with sociopolitical instability (SPI)

top graph in the figure shows that SPI is synchronously correlated with low β (reduced urban hierarchy) in city distributions for China. β recovers following the peaks in SPI, just as population does. The bottom graph in Figure 9.12 shows a 50-year lag between a high q/β ratio and rise in SPI, much as population growth relative to resources predicts a rise in SPI.

Conclusions

This study and those that preceded it began as experiments in building from two sets of sources, one in quantitative history and the work of Goldstone (1991), Turchin (2003), and Spufford (2002); and the other in more meaningful mathematical and measurement concepts that incorporate city-size distributions as an object of study. The development of consistent and asymptotically unbiased estimates of variations in city-size distributions using maximal likelihood (MLE) allowed a level of precision and accuracy – although we still need continuity corrections for very small samples – that led to useful findings in this study that are likely to be reliable.

By focusing on the 75 largest cities over a series of time periods that go back to antiquity – spaced closely enough to obtain the quantitative variations of the full cycles of city-system oscillations – Chandler (1987) made available a full run of data for studying how city-system evolution couples with agrarian socio-political dynamics. Initially, this will not be equally possible in every region at every time period, but only where the density of cities is sufficient for quantitative study. By focusing on Eurasia and its major regions, including China, we availed ourselves of some of the richest pockets of Chandler's comparative data on cities, especially for the early period of globalization, where China had the largest number of large cities. In refining the present model for comparison with other regions, we will consider whether to adjust for Chandler's possible underestimation of walled city populations for China, but the biasing assumption he made for China's walled cities (Appendix A) does not carry over to other regions. We did find strong evidence of historical periods of rise and fall in the city systems of different regions, and time-lagged effects of changes in city-size distributions in one region on other regions. These are weak and slow from Mid-Asia to China, and strong and fast from China to Europe, which makes sense in terms of the Silk Roads trade. This provides additional evidence of synchronies missing from Chase-Dunn et al. (2006) and the studies of Eurasian synchrony. Many of the correlations, however, are time-lagged rather than temporally synchronous. The effects run in the directions suggested by Modelski and Thompson (1996), e.g., China to Europe. Suggestions for improvement in methods and measurement and the possible impact of measurement biases on our results are given in Appendix B.

We are reasonably confident in concluding that the Pareto I and II (q-exponential) measures of city hierarchies through time, especially when used in combination, can provide a measurement paradigm of standardized methods and tests of replication in historical comparisons. The attractive features of the q-measure gained added benefit from the precision of our measurements using the MLE method.

The richness of the supporting data should logically take us next to Middle Asia, its subregions, and the larger world from CE 700 that embraced the rise of Islam, the Mongols' use of the Silk Roads, and development of new towns and cities on those routes that linked China and the rest of Middle Asia into a global system. Such a further study, modeled on this one, would include the role of the Indic subcontinent, and that of the Mongols (Barfield, 1989; Boyle, 1977) in trade and conquest; the Arab colonization of North Africa and Spain; and in feeding urban developments in the Mediterranean, Russia, and Europe.

When we compare our results, measurements, and mathematical models to those of Goldstone's (1991) studies of structural demography, or the studies of secular cycles by Nefedov (1999) and Turchin (2003, 2005), we find several novelties that separate our findings from that of the standard Lotka–Volterra oscillation model for historical fluctuations. Turchin (2003, 2004), for example, argues the Lotka–Voltera dynamic works optimally when one of the interactive variables (say, the population/resource ratio measure of scarcity and socio-political violence) is offset by one-quarter cycle. Our cycle of city-size oscillations might be two to four times as long as Turchin's secular cycles (J. S. Lee divides his 800-year periods into two periods of 400, suggesting an early growth of early forms of "empire" in a region, then a time of turbulence in the second period; then a new cycle of empire). It is possible that the city cycle operates at one or both these time-scales, and at the spatial scales of larger alternating civilizational networks of states and forms of empire.

Long city-size system oscillations of c.800 years would not be offset by a one quarter cycle but by one-eighth of a cycle, which is a long period of instability (vulnerable to conquest from the outside following internal instabilities).

From our perspective, however, socio-political instability is not smoothly cyclical but episodic. Rebellions, insurrections, and all sorts of protest are events that mobilize people in a given time and generation, and has impacts that, when repeated frequently, have massive effects. We see this in long-term correlations with SPI, such as internecine wars in China.

We have been able to discern some of the effects of trade fluctuations (if not trade network structure) in these models. The monetary liquidity variable for China, in one of our tests, showed the effect of a trade-related Silk Roads variable on q. We think that it is possible to reconstruct trade routes as a historical time series, and to perform an ordinal ranking of trade volumes on these routes. We think that these have strong effects – along with disruptive conflicts and political or empire boundaries – on the economies of individual cities and regions, and that these variables could be shown to have dynamical interactions within the context of secular cycles and the rise and fall of urban systems.

We have not performed a forward-looking prediction, but with MLE we expect to be able to make a better estimation, and possibly to correct the

biases in a reconstruction of Chandler's China dataset. Some of the patterns that we see in our data with regard to globalizing modernization are consistent with prior knowledge, but others are startling. The developmental trends of scale are to be expected – larger global cities; larger total urban population; and larger total population. We can also now investigate whether, with time, the parameter (Pareto II "scale" or σ) for crossover to the power-law diminishes, so that more and more of the city distribution becomes power law, and more consistent with much of the previous work on power-law scaling.

What is startling is that some long-wave oscillations in q are very long. Hopefully, a long-term trend and contemporary structure of Zipfian city distributions is an indicator of stability, but even the twentieth-century data indicate that instabilities are still very much present and thus likely to rest on historical contingencies (somewhat like the occurrence of a next earthquake larger than any seen in x years prior to it), and very much open to the effects of warfare and internal conflicts that are likely to be affected by population growth, and as opposed to the stabilization of trade benefiting per capita resources ratios.

The directions of change in q are largely predictable as a function of the current-state variables (such as population/resource ratios and socio-political violence) in the historical dynamics models up to, but not yet including, the contemporary period. It is not yet evident how to derive predictions for the contemporary era, given the new configurations of industrial societies, but it is very probable that the predictions that do emerge for the present will contain processes that have operated in the past.

On the issue of the coupling of cycles, Turchin cycles seem to embed two leading polity cycles (Modelski and Thompson, 1996) that average about 110 years. These are averages, and actual timings vary, but we have given explanations elsewhere for why these average cycle-lengths might tend to diminish by half as each embedded process tends to operate at successively smaller spatial scales. We hypothesize an embedding of dynamical processes that runs from trading zone network sizes and rise and fall of city-size distributions that cycle roughly 200, 400, or 800 years, partly dependent on the severity of the declines.

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Appendix A: Chandler's Chinese city data

Many Chinese historians question the estimates of Chinese city population densities given by Chandler (1987: 7): "Chinese cities tend to have an especially low density because of the Chinese refusal to sleep below anyone, so their houses are of nearly all of just one story. Hence, inland Chinese cities had a density of only about 75 per hectare, and even in seaports or the imperial capital the density hardly exceeded 100." One such anonymous reviewer considered this an underestimate by half, based on current knowledge.

To test the effects of these likely underestimates in population, we examined the data from the first of our periods, as shown in after checking Chandler's (1987: 417–51) text to note the basis for his estimates for each city. We adjusted the five city estimates affected, by raising each by 50 per cent (half the attributed error); recomputed the number of cities for each size category; and then compared the power-law coefficients for the original and the revised data as well as changes in the estimates of q. Changes were evident and significant in each case. They did not change the r2 for goodness of fit, but for q, for example, the value changed from 1.9 to 0.7. Similar tests can be performed for other periods, but we may need independent evidence with regard to the extent to which the critique merits new density computations.

Possible errors of this sort in the assumptions behind Chandler's estimation, then, could change our scaling results. These biases could change the patterns of variation in q seen in our results. Because Chandler uses common assumptions throughout these historical periods up to 1950, however, it is possible that they may not affect our historical comparisons about relative changes in q, at least up until 1950. For 1962, 1968, and 1970, he has taken the city population data of Richard Forstall of the company Rand McNally and Co. To test the robustness of our results, however, would entail a project that builds on the Microsoft ExcelTM spreadsheet for the Chandler data as exemplified above. By consulting his book to pick out each variable used to make multiplicative estimates – such as the size of urban area, the number of soldiers (e.g. times six), the number of streets, etc. – a spreadsheet of calculations with the multiples and their base numbers might be useful to improve Chandler's estimates using a Bayesian weighting by consistency.

Chandler (1987: 6) also notes that "the large growth of suburbs outside city walls had not begun before 1850 except in the newly rising industrial conurbations of Britain." This might cause problems "since the presence of suburbs has been well documented in history with new walls built to enclose a population that had spread beyond the original walls" (Pasciuti and Chase-Dunn, 2002: 1). Chandler does include suburbs in his criteria for city boundaries, however, and in his estimates throughout the book.

Appendix B: Suggestions toward improvement in methods

We began this study with Newman (2005) as a guide to power-law fitting and Borges (2004) as a guide to *q*-exponential fitting, and ended with the methods of Clauset et al. (2007) for the former and Shalizi (2007) for the latter. Physicists using these methods had not thought sufficiently about the relevance of a cutoff for the lower range at which *q*-exponential modeling has real-world limits (like size of smallest cities) as a problem in statistical estimation. We made useful discoveries about the need for such a cutoff for *q*-exponential distributions in addition to the usual crossover parameter; about the substantive meaning of such cutoffs; in their usefulness for direct comparison of Pareto and *q*-exponential MLE fits; and in the possibility of fitting both distributions in the same probability space with meaningful cutoffs and comparisons. Minimal xq_{min} cutoffs for *q*-exponential fitting, akin to those of Clauset et al. for the Pareto, are still needed in Shalizi's Pareto II approach to *q*-exponentials, and in Borges' maximum entropy approach.

Clauset et al.'s study also showed the need for new procedures to correct for small-sample bias within their framework of consistent and asymptotically unbiased maximal likelihood methods for estimating Pareto distribution parameters.[10] Our tests for small sample effects showed, for samples of twenty cities and under, non-significant correlations between estimates of the Pareto slope and sample size, with a tendency for smaller samples to overestimate Pareto slope. The lack of temerity with which we approached these problems in the face of small samples – where most researchers would be dissuaded to undertake our study – was balanced by our insistence on replication, in test after test, which showed considerable *non*-convergence of different methods of estimating *q* with small samples: Spss nonlinear regression, Excel solver minimization of error, methods of Newton-Raphson, and Shalizi's Pareto II MLE methods.

No one has studied as yet the small-sample bias in *q*-exponential parameter estimates with MLE or other methods, and the need for small-sample corrections for *q*. Our studies demonstrate this need: Our MLE estimates of *q* showed that smaller numbers of cities in single-period samples often tend to significantly overestimate *q*, which implies underestimating the asymptotic slope of the tail, running opposite to small sample MLE overestimation of Pareto slope. Whether this accounts for the nonconformal tendency we observed for Pareto slope estimates and *q*-distribution tails remains to be seen. These divergences may be indicators of city system distribution collapse, not inconsistent with our hypothesis testing results, but there may be better and more accurate ways to tell the story about non-Zipfian tendencies, Zipfian and *q*-exponential cutoffs, and historical dynamics.

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[1] Our special thanks go to Constantino Tsallis, the inspiration for this study, who patiently taught Doug and Nataša the fundamentals of *q*-exponential concepts and methods and then answered questions as they proceeded through the substantive analysis; and to Peter Turchin for generously providing data and suggestions for analysis. Any errors, however, remain our own. Thanks also to Robert McC. Adams, George Modelski, William Thompson, and Carter Butts for critical commentary and suggestions. Support from the EU project ISCOM, "Information Society as a Complex System," headed by David Lane, Sander van der Leeuw, Geoff West, and Denise Pumain, is acknowledged for the city-sizes project. We thank the leaders and members of the project for their critical commentary. The larger project, "Civilizations as Dynamic Networks," forms part of a Santa Fé Institute Working Group, for which SFI support is acknowledged. An early draft of the paper was presented to the Seminar on "Globalization as Evolutionary Process: Modeling, Simulating, and Forecasting Global Change," sponsored by the Calouste Gulbenkian Foundation, meeting at the International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria, 6–8 April, 2006. The reliability of the final analysis would not have been possible without the help of Cosma Shalizi, who derived and then programmed an MLE solution to the problems of estimating the parameters of historical city-size distributions, many of which had relatively small lists of largest city sizes once we got to the regional level. Because the current MLE methods are not only asymptotically normal and unbiased but also consistent, future research can incorporate a small-sample correction into the estimates of both the Pareto I and Pareto II fits to empirical distributions. Further research can also use the new methods of Clauset, Shalizi and Newman (2007) that provide likelihood ratios for evaluating relative log-likelihood of fit comparing Pareto I and II, lognormal, exponential, and stretched exponential distributions.

[2] Noting from the shared database that the top echelon of cities in a single region may be swept away in a short period by inter-regional competition, Batty (2006) refers to our work on instabilities at the level of city systems.

[3] For example, excluding two primate city outliers, the next-largest 16 cities for 1998 in the US (over 11 million) show a steep log–log slope, those ranking down to 0.5 million show a shallower slope, those to 0.1 million a much shallower slope, and then the power-law disappears altogether (Malacarne et al., 2001: 2).

[4] A typical batch of instructions, for example, might be:

```
china.900 <- c(500,150,90,81,75,75,70,65,60,58,49,47,40,40)
china.900.tsal.fit <- tsal.fit(china.900,xmin=40) # Assigns the results of the fit to the object
china.900.tsal.fit # Displays the estimated parameters and information about the fit
china.900.tsal.errors <- tsal.bootstrap.errors(china.900.tsal.fit,reps=100)
china.900.tsal.errors # Displays the bootstrapped error estimates.
```

[5] These include:

(a) The Yq estimates an expected "largest city size" M consistent with the body of the size distribution. This requires simultaneous estimation of M and Y(0) to solve $Y(0)\ P_{\theta,\sigma}\ (X \geq M) \approx M$.

(b) The total urban population can be estimated from Yq without having data on all smaller cities, although this feature is not utilized here.

(c) Equation (1) and Yq may be fitted without the largest city so as to derive from the model an expected size for the largest city.
(d) This gives our model a ratio measure of the largest city size to its expected size from Yq.
(e) Yq has a known derivative Y q (x) = Y0/κ[1 – (1 –q)x/κ](q/(1−q)), giving the slope of the curve Yq(x) for any city size x.

Solving for Yq(M) ≈ M for the estimated largest city size M consistent with Yq, gives Y q(M) as the slope of the Yq at M, and converges with β =1/(q−1) for the Pareto power-law slope when $1 < q \leq 2$.

[6] Commentary on the White et al. (2004) paper on the present topic elicited the suggestion from Cosma Shalizi that we should consider MLE estimates for fitting q-exponentials to city-size data. Given our small sample sizes from the Chandler data as we moved from a world sample (75 data points) to regional samples, we called on Shalizi in late 2006 to derive the MLE equation for us, which he did. He then found earlier derivations such as those of Arnold (1983) and others. We are greatly indebted to Shalizi for writing functions in R to make MLE estimates of the q-exponential parameters and bootstrap estimates of the standard errors of these estimates.

[7] If we remove the σ«1 cases for China, for example, the relationship expected of our estimates, namely, (–)β ≈ 1/(1−q), is closely fulfilled where $1 < q \leq 2$, but is contradicted where σ «1. With the improved methods of Clauset, Shalizi, and Newman (2007), and working with Clauset and Shalizi, we were able to determine two additional parameters which we call xq_{min} and xp_{min} for the Pareto II and Pareto I, respectively, which are the lower cutoffs in size that maximize the MLE fit of each distribution. Using their data on populations of US cities in the 2000 US Census, we were able to determine that xq_{min} = 5,000 for best fit of the q-exponential, which also closely approximates the sizes of smallest cities found archeologically in different times and regions, while they report xp_{min} ~ 52,000 as a cutoff for the best MLE fit of a power-law to these same data (Clauset, Shalizi, and Newman 2007). This later finding, made just before proofreading this chapter, supports the approach of the current study, but also points us towards methods that can be further improved. Having an MLE solution for xq_{min} also entails the possibility of an MLE solution to estimating Y0 in the equation for the q-exponential, that is, an estimation of the total urban population consistent with the q-exponential shape of a city-side distribution.

[8] When σ «1 values are not correlated with small sample sizes, it is a signal to investigate the possibility that the region of aggregation is not properly specified. Having a rule-of-thumb for regional boundary misspecification will be helpful in further research in this and other areas of study.

[9] Elsewhere, we explore estimates of the total urban population asymptote, Y(0), from fitting the Pareto II and equivalent q-exponential distributions.

[10] Pareto MLE fits by Clauset et al.'s methods have a small-sample bias with greater overestimates of the Pareto slope the smaller the sample, consistent with our empirical finding. This can be adjusted by a small-sample correction but that has been done or programmed as yet and so we have not been able to use correction methods.